



The electromagnetic dark sector

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ARTICLE INFO

Article history:

Received 16 September 2009

Received in revised form 20 January 2010

Accepted 12 February 2010

Available online 19 February 2010

Editor: L. Alvarez-Gaumé

Keywords:

Quantum fields in curved space–time

Dark energy

ABSTRACT

We consider electromagnetic field quantization in an expanding universe. We find that the covariant (Gupta–Bleuler) method exhibits certain difficulties when trying to impose the quantum Lorenz condition on cosmological scales. We thus explore the possibility of consistently quantizing without imposing such a condition. In this case there are three physical states, which are the two transverse polarizations of the massless photon and a new massless scalar mode coming from the temporal and longitudinal components of the electromagnetic field. An explicit example in de Sitter space–time shows that it is still possible to eliminate the negative norm state and to ensure the positivity of the energy in this theory. The new state is decoupled from the conserved electromagnetic currents, but is non-conformally coupled to gravity and therefore can be excited from vacuum fluctuations by the expanding background. The cosmological evolution ensures that the new state modifies Maxwell's equations in a totally negligible way on sub-Hubble scales. However, on cosmological scales it can give rise to a non-negligible energy density which could explain in a natural way the present phase of accelerated expansion of the universe.

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1. Introduction

The fact that the present phase of accelerated expansion of the universe [1] finds no natural explanation within known physics suggests the possibility that General Relativity could be inappropriate to describe gravity on cosmological scales. Nevertheless, despite the many attempts to find modified gravity theories with late time accelerated solutions [2,3], the models considered so far are generally plagued by classical or quantum instabilities, fine-tuning problems or local gravity inconsistencies.

However, apart from gravity, there is another long-range interaction which could become relevant on cosmological scales, which is nothing but electromagnetism. It is generally assumed that due to the strict electric neutrality of the universe on large scales, the only relevant interaction in cosmology is gravitation. However, this assumption seems to conflict with observations which indicate that relatively strong magnetic fields with very large coherence lengths are present in galaxies and galaxy clusters, and whose origin is still unknown [4].

In this work, we consider (quantum) electromagnetic fields in an expanding universe. Unlike previous works, we will not follow the standard Coulomb gauge approach, but instead we will concentrate on the covariant quantization method. This method is found to exhibit certain difficulties when trying to impose the quantum Lorenz condition on cosmological scales [5]. In order to overcome

these problems, and instead of introducing ghosts fields, we explore the possibility of quantizing without imposing any subsidiary condition. This implies that the electromagnetic field would contain an additional (scalar) polarization, similar to that generated in the massless limit of massive electrodynamics [6]. Interestingly, the energy density of quantum fluctuations of this new electromagnetic state generated during inflation gets frozen on cosmological scales, giving rise to an effective cosmological constant [7].

2. Electromagnetic quantization in flat space–time

Let us start by briefly reviewing electromagnetic quantization in Minkowski space–time [8]. The action of the theory reads:

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu \right) \quad (1)$$

where J_μ is a conserved current. This action is invariant under gauge transformations $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$ with Λ an arbitrary function of space–time coordinates. At the classical level, this action gives rise to the well-known Maxwell's equations:

$$\partial_\nu F^{\mu\nu} = J^\mu. \quad (2)$$

However, when trying to quantize the theory, several problems arise related to the redundancy in the description due to the gauge invariance. Thus in particular, we find that it is not possible to construct a propagator for the A_μ field and also that “unphysical” polarizations of the photon field are present. Two different approaches are usually followed in order to avoid these difficulties.

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In the first one, which is the basis of the Coulomb gauge quantization, the gauge invariance of the action (1) is used to eliminate the “unphysical” degrees of freedom. With that purpose the (Lorenz) condition $\partial_\mu A^\mu = 0$ is imposed by means of a suitable gauge transformation. Thus, the equations of motion reduce to:

$$\square A_\mu = J_\mu. \quad (3)$$

The Lorenz condition does not fix completely the gauge freedom, still it is possible to perform residual gauge transformations $A_\mu \rightarrow A_\mu + \partial_\mu \theta$, provided $\square \theta = 0$. Using this residual symmetry and taking into account the form of Eq. (3), it is possible to eliminate one additional component of the A_μ field in the asymptotically free regions (typically A_0) which means $\vec{\nabla} \cdot \vec{A} = 0$, so that finally the temporal and longitudinal photons are removed and we are left with the two transverse polarizations of the massless free photon, which are the only modes (with positive energies) which are quantized in this formalism.

The second approach is the basis of the covariant (Gupta-Bleuler) and path-integral formalisms. The starting point is a modification of the action in (1), namely:

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + A_\mu J^\mu \right). \quad (4)$$

This action is no longer invariant under general gauge transformations, but only under residual ones. The equations of motion obtained from this action now read:

$$\partial_\nu F^{\mu\nu} + \xi \partial^\mu (\partial_\nu A^\nu) = J^\mu. \quad (5)$$

In order to recover Maxwell's equation, the Lorenz condition must be imposed so that the ξ term disappears. At the classical level this can be achieved by means of appropriate boundary conditions on the field. Indeed, taking the four-divergence of the above equation, we find:

$$\square (\partial_\nu A^\nu) = 0 \quad (6)$$

where we have made use of current conservation. This means that the field $\partial_\nu A^\nu$ evolves as a free scalar field, so that if it vanishes for large $|t|$, it will vanish at all times. At the quantum level, the Lorenz condition cannot be imposed as an operator identity, but only in the weak sense $\partial_\nu A^{\nu(+)}|\phi\rangle = 0$, where $(+)$ denotes the positive frequency part of the operator and $|\phi\rangle$ is a physical state. This condition is equivalent to imposing $[\mathbf{a}_0(k) + \mathbf{a}_\parallel(k)]|\phi\rangle = 0$, with \mathbf{a}_0 and \mathbf{a}_\parallel the annihilation operators corresponding to temporal and longitudinal electromagnetic states. Thus, in the covariant formalism, the physical states contain the same number of temporal and longitudinal photons, so that their energy densities, having opposite signs, cancel each other.

Thus we see that also in this case, the Lorenz condition seems to be essential in order to recover standard Maxwell's equations and get rid of the negative energy states.

3. Covariant quantization in an expanding universe

So far we have only considered Maxwell's theory in flat space-time, however when we move to a curved background, and in particular to an expanding universe, then consistently imposing the Lorenz condition in the covariant formalism turns out to be difficult to realize. Indeed, let us consider the curved space-time version of action (4):

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right]. \quad (7)$$

Now the modified Maxwell's equations read:

$$\nabla_\nu F^{\mu\nu} + \xi \nabla^\mu (\nabla_\nu A^\nu) = J^\mu \quad (8)$$

and taking again the four divergence, we get:

$$\square (\nabla_\nu A^\nu) = 0. \quad (9)$$

We see that once again $\nabla_\nu A^\nu$ behaves as a scalar field which is decoupled from the conserved electromagnetic currents, but it is non-conformally coupled to gravity. This means that, unlike the flat space-time case, this field can be excited from quantum vacuum fluctuations by the expanding background in a completely analogous way to the inflaton fluctuations during inflation. Thus this poses the question of the validity of the Lorenz condition at all times.

In order to illustrate this effect, we will present a toy example. Let us consider quantization in the absence of currents, in a spatially flat expanding background, whose metric is written in conformal time as $ds^2 = a(\eta)^2(d\eta^2 - d\vec{x}^2)$ with $a(\eta) = 2 + \tanh(\eta/\eta_0)$ where η_0 is constant. This metric possess two asymptotically Minkowskian regions in the remote past and far future. We solve the coupled system of Eq. (8) for the corresponding Fourier modes of the conformal field $\mathcal{A}_\mu = (aA_0, \vec{A})$, which are defined as $\mathcal{A}_\mu(\eta, \vec{x}) = \int d^3k \mathcal{A}_{\mu\vec{k}}(\eta) e^{i\vec{k}\vec{x}}$. Thus, for a given mode \vec{k} , the \mathcal{A}_μ field is decomposed into temporal, longitudinal and transverse components. The corresponding equations read:

$$\begin{aligned} \mathcal{A}_{0\vec{k}}'' - \left[\frac{k^2}{\xi} - 2\mathcal{H}' + 4\mathcal{H}^2 \right] \mathcal{A}_{0\vec{k}} - 2ik \left[\frac{1+\xi}{2\xi} \mathcal{A}_{\parallel\vec{k}}' - \mathcal{H} \mathcal{A}_{\parallel\vec{k}} \right] &= 0, \\ \mathcal{A}_{\parallel\vec{k}}'' - k^2 \xi \mathcal{A}_{\parallel\vec{k}} - 2ik\xi \left[\frac{1+\xi}{2\xi} \mathcal{A}_{0\vec{k}}' + \mathcal{H} \mathcal{A}_{0\vec{k}} \right] &= 0, \\ \vec{\mathcal{A}}_{\perp\vec{k}}'' + k^2 \vec{\mathcal{A}}_{\perp\vec{k}} &= 0 \end{aligned} \quad (10)$$

with $\mathcal{H} = a'/a$ and $k = |\vec{k}|$. We see that the transverse modes are decoupled from the background, whereas the temporal and longitudinal ones are non-trivially coupled to each other and to gravity. Let us prepare our system in an initial state $|\phi\rangle$ belonging to the physical Hilbert space, i.e. satisfying $\partial_\nu \mathcal{A}_{in}^{\nu(+)}|\phi\rangle = 0$ in the initial flat region. Because of the expansion of the universe, the positive frequency modes in the *in* region with a given temporal or longitudinal polarization λ will become a linear superposition of positive and negative frequency modes in the *out* region and with different polarizations λ' (we will work in the Feynman gauge $\xi = -1$), thus we have:

$$\mathcal{A}_{\mu\vec{k}}^{\lambda(in)} = \sum_{\lambda'=0,\parallel} [\alpha_{\lambda\lambda'}(\vec{k}) \mathcal{A}_{\mu\vec{k}}^{\lambda'(out)} + \beta_{\lambda\lambda'}(\vec{k}) \overline{\mathcal{A}}_{\mu-\vec{k}}^{\lambda'(out)}] \quad (11)$$

or in terms of creation and annihilation operators:

$$\mathbf{a}_{\lambda'}^{(out)}(\vec{k}) = \sum_{\lambda=0,\parallel} [\alpha_{\lambda\lambda'}(\vec{k}) \mathbf{a}_\lambda^{(in)}(\vec{k}) + \overline{\beta_{\lambda\lambda'}(\vec{k})} \mathbf{a}_\lambda^{(in)\dagger}(-\vec{k})] \quad (12)$$

with $\lambda, \lambda' = 0, \parallel$ and where $\alpha_{\lambda\lambda'}$ and $\beta_{\lambda\lambda'}$ are the so-called Bogoliubov coefficients (see [9] for a detailed discussion), which are normalized in our case according to:

$$\sum_{\rho, \rho'=0,\parallel} (\alpha_{\lambda\rho} \overline{\alpha_{\lambda'\rho'}} \eta_{\rho\rho'} - \beta_{\lambda\rho} \overline{\beta_{\lambda'\rho'}} \eta_{\rho\rho'}) = \eta_{\lambda\lambda'} \quad (13)$$

with $\eta_{\lambda\lambda'} = \text{diag}(-1, 1)$ with $\lambda, \lambda' = 0, \parallel$. Notice that the normalization is different from the standard one [9], because of the presence of negative norm states.

Thus, the system will end up in a final state which no longer satisfies the weak Lorenz condition i.e. in the *out* region $\partial_\nu \mathcal{A}_{out}^{\nu(+)}|\phi\rangle \neq 0$. This is shown in Fig. 1, where we have computed the final number of temporal and longitudinal photons

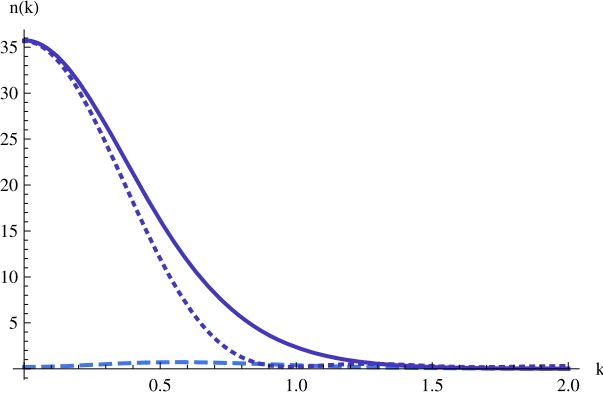


Fig. 1. Occupation numbers for temporal (continuous line) and longitudinal (dashed line) photons in the *out* region vs. k in η_0^{-1} units. We also show the analytical approximate solution (dotted line) given in the main text.

$n_{\lambda'}^{out}(k) = \sum_{\lambda} |\beta_{\lambda\lambda'}(\vec{k})|^2$, starting from an initial vacuum state with $n_0^{in}(k) = n_{||}^{in}(k) = 0$. We see that, as commented above, in the final region $n_0^{out}(k) \neq n_{||}^{out}(k)$ and the state no longer satisfies the Lorenz condition. Notice that the failure comes essentially from large scales ($k\eta_0 \ll 1$), since on small scales ($k\eta_0 \gg 1$), the Lorenz condition can be restored. This can be easily interpreted from the fact that on small scales the geometry can be considered as essentially Minkowskian.

We can obtain an approximate analytical solution for the Bogoliubov coefficients by assuming an expansion in which \mathcal{H} vanishes in the *in* and *out* regions, but it takes a constant value h in the period $(-t_0, t_0)$. Moreover, we shall neglect the production of longitudinal modes (which is justified from our numerical computation). For this simplified problem, the relevant Bogoliubov coefficients are given by:

$$\begin{aligned} \beta_{||0} &= \frac{1}{2a_{out}^2} e^{2t_0(h+ik)} \left[\left(1 - \frac{ik}{h}\right) \cosh \omega + \frac{2h - ik - \frac{k^2}{h}}{\sqrt{4h^2 - k^2}} \sinh \omega \right] \\ &\quad - \frac{1}{2a_{out}^2} \left(1 - \frac{ik}{h}\right) e^{2t_0h}, \\ \beta_{00} &= -\frac{2he^{4t_0h}}{a_{out}^2 \sqrt{4h^2 - k^2}} \sinh \omega \end{aligned} \quad (14)$$

where $\omega = 2t_0\sqrt{4h^2 - k^2}$, a_{out} is the scale factor in the *out* region and we have assumed that $a_{in} = 1$. In Fig. 1 we show the corresponding production of temporal modes $n_0^{out}(k) = |\beta_{00}|^2 + |\beta_{||0}|^2$ for $h = 0.29$ and $t_0 = 2$ (which approximately corresponds to the Hubble expansion rate used for the numerical computation). Notice that the approximate analytical solution reproduces very accurately the numerical result at small k . This is because, on that region, the production of longitudinal modes is practically zero and for very large wavelengths, the production of temporal states is not sensitive to the particular shape of the expansion rate.

These results show that in a space-time configuration with asymptotic flat regions, an initial state satisfying the weak Lorenz condition does not necessarily satisfy it at a later time. In order to overcome this problem, it is possible to impose a more stringent gauge-fixing condition. Thus, following [10] we can define the physical states $|\phi\rangle$ as those such that $\nabla_\mu A^{\mu(+)}|\phi\rangle = 0$, $\forall \eta$. Although this is a perfectly consistent solution, notice that the separation in positive and negative frequency parts depends on the space-time geometry and therefore, the determination of the physical states requires a previous knowledge of the geometry of the universe at all times.

Another possible way out would be to modify the standard Gupta–Bleuler formalism by including ghosts fields as done in non-Abelian gauge theories [11]. With that purpose, the action of the theory (7) can be modified by including the ghost term (see [12]):

$$S_g = \int d^4x \sqrt{g} g^{\mu\nu} \partial_\mu \bar{c} \partial_\nu c \quad (15)$$

where c are the complex scalar ghost fields. It is a well-known result [12,13] that by choosing appropriate boundary conditions for the electromagnetic and ghosts Green's functions, it is possible to get $\langle \phi | T_{\mu\nu}^\xi + T_{\mu\nu}^g | \phi \rangle = 0$, where $T_{\mu\nu}^\xi$ and $T_{\mu\nu}^g$ denote the contribution to the energy-momentum tensor from the ξ term in (7) and from the ghost term (15) respectively. Notice that a choice of boundary conditions in the Green's functions corresponds to a choice of vacuum state. Therefore, also in this case an a priori knowledge of the future behaviour of the universe geometry is required in order to determine the physical states.

In this work we follow a different approach in order to deal with the difficulties found in the Gupta–Bleuler formalism and we will explore the possibility of quantizing electromagnetism in an expanding universe without imposing any subsidiary condition.

4. Quantization without the Lorenz condition

In the previous section it has been shown that although the Lorenz gauge-fixing conditions can be imposed in the covariant formalism, this cannot be done in a straightforward way. These difficulties could be suggesting some more fundamental obstacle in the formulation of an electromagnetic gauge invariant theory in an expanding universe. As a matter of fact, electromagnetic models which break gauge invariance on cosmological scales have been widely considered in the context of generation of primordial magnetic fields (see, for instance, [14]).

Let us then explore the possibility that the fundamental theory of electromagnetism is not given by the gauge invariant action (1), but by the gauge non-invariant action (7). Notice that although (7) is not invariant under general gauge transformations, it respects the invariance under residual ones and, as shown below, in Minkowski space-time, the theory is completely equivalent to standard QED. Since the fundamental electromagnetic theory is assumed non-invariant under arbitrary gauge transformations, then there is no need to impose the Lorenz constraint in the quantization procedure. Therefore, having removed one constraint, the theory contains one additional degree of freedom. Thus, the general solution for the modified equations (8) can be written as:

$$A_\mu = \mathcal{A}_\mu^{(1)} + \mathcal{A}_\mu^{(2)} + \mathcal{A}_\mu^{(s)} + \partial_\mu \theta \quad (16)$$

where $\mathcal{A}_\mu^{(i)}$ with $i = 1, 2$ are the two transverse modes of the massless photon, $\mathcal{A}_\mu^{(s)}$ is the new scalar state, which is the mode that would have been eliminated if we had imposed the Lorenz condition and, finally, $\partial_\mu \theta$ is a purely residual gauge mode, which can be eliminated by means of a residual gauge transformation in the asymptotically free regions, in a completely analogous way to the elimination of the A_0 component in the Coulomb quantization. The fact that Maxwell's electromagnetism could contain an additional scalar mode decoupled from electromagnetic currents, but with non-vanishing gravitational interactions, was already noticed in a different context in [6].

In order to quantize the free theory, we perform the mode expansion of the field with the corresponding creation and annihilation operators for the *three* physical states:

$$A_\mu = \int d^3\vec{k} \sum_{\lambda=1,2,s} [\mathbf{a}_\lambda(k) \mathcal{A}_{\mu k}^{(\lambda)} + \mathbf{a}_\lambda^\dagger(k) \overline{\mathcal{A}_{\mu k}^{(\lambda)}}] \quad (17)$$

where the modes are required to be orthonormal with respect to the scalar product (see for instance [10]):

$$\begin{aligned} (\mathcal{A}_k^{(\lambda)}, \mathcal{A}_{k'}^{(\lambda')}) &= i \int_{\Sigma} d\Sigma_{\mu} [\overline{\mathcal{A}_{\nu k}^{(\lambda)}} \Pi_{k'}^{(\lambda')\mu\nu} - \overline{\Pi_k^{(\lambda)\mu\nu}} \mathcal{A}_{\nu k'}^{(\lambda')}] \\ &= \delta_{\lambda\lambda'} \delta^{(3)}(\vec{k} - \vec{k}'), \quad \lambda, \lambda' = 1, 2, s \end{aligned} \quad (18)$$

where $d\Sigma_{\mu}$ is the three-volume element of the Cauchy hypersurfaces. In a Robertson–Walker metric in conformal time, it reads $d\Sigma_{\mu} = a^4(\eta)(d^3x, 0, 0, 0)$. The generalized conjugate momenta are defined as:

$$\Pi^{\mu\nu} = -(F^{\mu\nu} - \xi g^{\mu\nu} \nabla_{\rho} A^{\rho}). \quad (19)$$

Notice that the three modes can be chosen to have positive normalization. The equal-time commutation relations:

$$[\mathcal{A}_{\mu}(\eta, \vec{x}), \mathcal{A}_{\nu}(\eta, \vec{x}')] = [\Pi^{0\mu}(\eta, \vec{x}), \Pi^{0\nu}(\eta, \vec{x}')] = 0 \quad (20)$$

and

$$[\mathcal{A}_{\mu}(\eta, \vec{x}), \Pi^{0\nu}(\eta, \vec{x}')] = i \frac{\delta_{\mu}^{\nu}}{\sqrt{g}} \delta^{(3)}(\vec{x} - \vec{x}') \quad (21)$$

can be seen to imply the canonical commutation relations:

$$[\mathbf{a}_{\lambda}(\vec{k}), \mathbf{a}_{\lambda'}^{\dagger}(\vec{k}')] = \delta_{\lambda\lambda'} \delta^{(3)}(\vec{k} - \vec{k}'), \quad \lambda, \lambda' = 1, 2, s \quad (22)$$

by means of the normalization condition in (18). Notice that the sign of the commutators is positive for the three physical states, i.e. there are no negative norm states in the theory, which in turn implies that there are no negative energy states as we will see below in an explicit example.

Since $\nabla_{\mu} A^{\mu}$ evolves as a minimally coupled scalar field, as shown in (9), on sub-Hubble scales ($|k\eta| \gg 1$), we find that for arbitrary background evolution, $|\nabla_{\mu} A_k^{(s)\mu}| \propto a^{-1}$, i.e. the field is suppressed by the universe expansion, thus effectively recovering the Lorenz condition on small scales. Notice that this is a consequence of the cosmological evolution, not being imposed as a boundary condition as in the flat space-time case.

On the other hand, on super-Hubble scales ($|k\eta| \ll 1$), $|\nabla_{\mu} A_k^{(s)\mu}| = \text{const.}$ which, as shown in [7], implies that the field contributes as a cosmological constant in (7). Indeed, the energy-momentum tensor derived from (7) reads:

$$\begin{aligned} T_{\mu\nu} &= -F_{\mu\alpha} F_{\nu}^{\alpha} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \\ &+ \frac{\xi}{2} [g_{\mu\nu} ((\nabla_{\alpha} A^{\alpha})^2 + 2A^{\alpha} \nabla_{\alpha} (\nabla_{\beta} A^{\beta})) \\ &- 4A_{(\mu} \nabla_{\nu)} (\nabla_{\alpha} A^{\alpha})]. \end{aligned} \quad (23)$$

Notice that for the scalar electromagnetic mode in the super-Hubble limit, the contributions involving $F_{\mu\nu}$ vanish and only the piece proportional to ξ is relevant. Thus, it can be easily seen that, since in this case $\nabla_{\alpha} A^{\alpha} = \text{const.}$, the energy-momentum tensor is just given by:

$$T_{\mu\nu} = \frac{\xi}{2} g_{\mu\nu} (\nabla_{\alpha} A^{\alpha})^2 \quad (24)$$

which is the energy-momentum tensor of a cosmological constant. Notice that, as seen in (9), the new scalar mode is a massless free field. This is one of the most relevant aspects of the present model in which, unlike existing dark energy theories based on scalar fields, dark energy can be generated without including any potential term or dimensional constant.

Since, as shown above, the field amplitude remains frozen on super-Hubble scales and starts decaying once the mode enters the

horizon in the radiation or matter eras, the effect of the ξ term in (8) is completely negligible on sub-Hubble scales, since the initial amplitude generated during inflation is very small as we will show below. Thus, below 1.3 AU, which is the largest distance scale at which electromagnetism has been tested [15], the modified Maxwell's equations (8) are physically indistinguishable from the flat space-time ones (2).

Notice that in Minkowski space-time, the theory (7) is completely equivalent to standard QED. This is so because, although non-gauge invariant, the corresponding effective action is equivalent to the standard BRS invariant effective action of QED. Thus, the effective action for QED obtained from (2) by the standard gauge-fixing procedure reads:

$$\begin{aligned} e^{iW} &= \int [dA][dc][d\bar{c}][d\psi][d\bar{\psi}] \\ &\times e^{i \int d^4x (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_{\mu} A^{\mu})^2 + \eta^{\mu\nu} \partial_{\mu} \bar{c} \partial_{\nu} c + \mathcal{L}_F)} \end{aligned} \quad (25)$$

where \mathcal{L}_F is the Lagrangian density of charged fermions. The ξ term and the ghosts field appear in the Faddeev–Popov procedure when selecting an element of each gauge orbit. However, ghosts being decoupled from the electromagnetic currents can be integrated out in flat space-time, so that up to an irrelevant normalization constant we find:

$$e^{iW} = \int [dA][d\psi][d\bar{\psi}] e^{i \int d^4x (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_{\mu} A^{\mu})^2 + \mathcal{L}_F)} \quad (26)$$

which is nothing but the effective action coming from the gauge non-invariant theory (7) in flat space-time, in which no gauge-fixing procedure is required.

5. An explicit example: quantization in de Sitter space-time

Let us consider an explicit example which is given by the quantization in an inflationary de Sitter space-time with $a(\eta) = -1/(H_I \eta)$. Here we will take $\xi = 1/3$, although similar results can be obtained for any $\xi > 0$. For this case, in which $\mathcal{H}' = \mathcal{H}^2$, we can obtain the following expression for the temporal component in terms of the longitudinal one from (10):

$$\begin{aligned} \mathcal{A}_{0k} &= \frac{-i}{2k(4k^2 + 3\mathcal{H}^2)} \\ &\times \left[2 \frac{d^3 \mathcal{A}_{\parallel k}}{d\eta^3} - \mathcal{H} \frac{d^2 \mathcal{A}_{\parallel k}}{d\eta^2} + 10k^2 \frac{d \mathcal{A}_{\parallel k}}{d\eta} - 5\mathcal{H} k^2 \mathcal{A}_{\parallel k} \right]. \end{aligned} \quad (27)$$

On the other hand, if we insert this relation into the equation for $\mathcal{A}_{\parallel k}$ we obtain a fourth-order equation for the longitudinal component which is completely decoupled from \mathcal{A}_{0k} . This fourth-order equation can be expressed as follows:

$$\begin{aligned} \left[\frac{d^2}{d\eta^2} - 4 \frac{2k^2 + 3\mathcal{H}^2}{4k^2 + 3\mathcal{H}^2} \mathcal{H} \frac{d}{d\eta} + \frac{4k^4 + 6\mathcal{H}^4 - 9k^2 \mathcal{H}^2}{4k^2 + 3\mathcal{H}^2} \right] \\ \times \left[\frac{d^2}{d\eta^2} + 2\mathcal{H} \frac{d}{d\eta} + k^2 \right] \mathcal{A}_{\parallel k} = 0, \end{aligned} \quad (28)$$

where we have performed a factorization in such a way that the second bracket is nothing but the operator determining the evolution of a free scalar field, i.e., $\square\theta = 0$. Now, we shall use the notation $\mathcal{F}(\mathcal{G}(\mathcal{A}_{\parallel k})) = 0$, where \mathcal{F} and \mathcal{G} are the operators given in the first and second brackets in (28) respectively. In particular, $\mathcal{G} = \square$ as we have already said. Thus, in order to solve the equation we have to obtain the kernel of \mathcal{F} , whose solutions will be denoted by S , and, then, solve the equation for a free scalar field with S as an external source: $\mathcal{G}(\mathcal{A}_{\parallel k}) = S$. This method has the advantage of allowing to identify the pure residual gauge mode as

the homogeneous solution of this last equation. The explicit solution for the normalized scalar state is:

$$\begin{aligned}\mathcal{A}_{0k}^{(s)} &= -\frac{1}{(2\pi)^{3/2}} \frac{i}{\sqrt{2k}} \left\{ k\eta e^{-ik\eta} \right. \\ &\quad \left. + \frac{1}{k\eta} \left[\frac{1}{2}(1+ik\eta)e^{-ik\eta} - k^2\eta^2 e^{ik\eta} E_1(2ik\eta) \right] \right\} e^{i\vec{k}\vec{x}}, \\ \mathcal{A}_{\parallel k}^{(s)} &= \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k}} \left\{ (1+ik\eta)e^{-ik\eta} \right. \\ &\quad \left. - \left[\frac{3}{2}e^{-ik\eta} + (1-ik\eta)e^{ik\eta} E_1(2ik\eta) \right] \right\} e^{i\vec{k}\vec{x}}\end{aligned}\quad (29)$$

where $E_1(x) = \int_1^\infty e^{-tx}/t dt$ is the exponential integral function and we have fixed the integration constants so that the mode is canonically normalized according to (18). Using this solution, we find:

$$\nabla_\mu \mathcal{A}_k^{(s)\mu} = -\frac{a^{-2}(\eta)}{(2\pi)^{3/2}} \frac{ik}{\sqrt{2k}} \frac{3}{2} \frac{(1+ik\eta)}{k^2\eta^2} e^{-ik\eta+i\vec{k}\vec{x}} \quad (30)$$

so that the field is suppressed in the sub-Hubble limit as $\nabla_\mu \mathcal{A}_k^{(s)\mu} \sim \mathcal{O}((k\eta)^{-2})$.

On the other hand, from the energy density given by $\rho_A = T_0^0$, we obtain in the sub-Hubble limit the corresponding Hamiltonian, which is given by:

$$H = \frac{1}{2} \int \frac{d^3\vec{k}}{a^4(\eta)} k \sum_{\lambda=1,2,s} [\mathbf{a}_\lambda^\dagger(\vec{k}) \mathbf{a}_\lambda(\vec{k}) + \mathbf{a}_\lambda(\vec{k}) \mathbf{a}_\lambda^\dagger(\vec{k})]. \quad (31)$$

We see that the theory does not contain negative energy states (ghosts). In fact, as shown in [7,16], the theory does not exhibit either local gravity inconsistencies or classical instabilities.

Finally, from (29) it is possible to obtain the dispersion of the effective cosmological constant during inflation:

$$\langle 0 | (\nabla_\mu \mathcal{A}^\mu)^2 | 0 \rangle = \int \frac{dk}{k} P_A(k) \quad (32)$$

with $P_A(k) = 4\pi k^3 |\nabla_\mu \mathcal{A}_k^{(s)\mu}|^2$. In the super-Hubble limit, we obtain for the power-spectrum:

$$P_A(k) = \frac{9H_I^4}{16\pi^2}, \quad (33)$$

in agreement with [7]. Notice that this result implies that $\rho_A \sim (H_I)^4$. The measured value of the cosmological constant then requires $H_I \sim 10^{-3}$ eV, which corresponds to an inflationary scale $M_I \sim 1$ TeV. Thus we see that the cosmological constant scale can be naturally explained in terms of physics at the electroweak scale.

Once we have computed the initial amplitude of the electromagnetic fluctuations, we can estimate the magnitude of the corrections induced by the new terms in the modified Maxwell's equations. Notice that (8) can be rewritten as the ordinary Maxwell's equation with an additional conserved current source:

$$\nabla_\nu F^{\mu\nu} = J^\mu + J_\xi^\mu \quad (34)$$

where $J_\xi^\mu = -\xi \nabla^\mu (\nabla_\nu A^\nu)$ satisfies $\nabla_\mu J_\xi^\mu = 0$ by virtue of (9). For sub-Hubble modes below 1.3 AU, which is the region on which electromagnetism has been tested, consistency requires the new current to be negligible. It is possible to estimate the present value of such a current by noticing that a Fourier mode k entered the Hubble radius in the radiation dominated era when the scale factor was $a_{in} \simeq H_0 a_{eq}^{1/2}/k$, with a_{eq} the scale factor at matter-radiation equality and where we have taken that today $a_0 = 1$. Therefore, since the initial amplitude $\nabla_\mu A^\mu \simeq H_I^2$ remains constant until

horizon reentry, today we obtain for the effective electric charge density created by the mentioned modes:

$$J_\xi^0 \sim k H_I^2 a_{in} \sim H_0 H_I^2 a_{eq}^{1/2} \sim 10^{-41} \text{ eV}^3 \quad (35)$$

which is independent of k and completely negligible since it roughly corresponds to the charge density of one electron in the volume of the Earth.

6. Conclusions

In this work we have considered the difficulties in the application of the covariant quantization method for electromagnetic fields in an expanding universe. Although the Lorenz gauge-fixing condition can in principle be formally applied also in cosmological contexts, we have explored the alternative possibility of quantization without subsidiary conditions, and therefore, without the introduction of ghost fields.

In this approach the theory is invariant only under residual gauge transformations and a new scalar mode appears for the electromagnetic field. Despite these facts, standard QED is recovered in the flat space-time limit as the new scalar state is completely decoupled from the conserved electromagnetic currents (only transverse photons couple to them [17]). The residual gauge symmetry of the theory ensures that the negative norm state can be eliminated and that the energy is positive as shown in an explicit example.

This quantization procedure is found to have interesting consequences for dark energy. Thus, the energy density of the new mode on super-Hubble scales (which is essentially the temporal part of the electromagnetic field) is seen to behave as a cosmological constant irrespectively of the expansion rate of the universe. The new mode can be generated during inflation, in a similar way to the inflaton fluctuations and this fact allows to establish a link between the scale of inflation and the value of the cosmological constant.

Acknowledgements

This work has been supported by Ministerio de Ciencia e Innovación (Spain) project numbers FIS 2008-01323 and FPA 2008-00592, UCM-Santander PR34/07-15875, CAM/UCM 910309 and MEC grant BES-2006-12059.

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